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Vector and Scalar Meson Resonances in $K \to \pi\pi\pi$ Decays

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ABSTRACT

Corrections to $K \to \pi\pi\pi$ decays induced by vector and scalar meson exchange are investigated within chiral perturbation theory. The widths of scalar mesons are analyzed and their influence on $K \to \pi\pi\pi$ parameters were examined. The overall corrections were found to be parameter dependent, but contributing in some cases as much as 10%.

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1.Introduction

The Chiral Perturbation Theory (CHPT) offers a successful scheme for description of strong, weak and electromagnetic interactions at low energies [1, 2, 3, 4, 5, 6]. The light pseudoscalar mesons play a role of Goldstone bosons of the $SU(3)_L \times SU(3)_R$ symmetry and transition amplitudes expanded in powers of meson momenta and masses can be calculated using phenomenological lagrangians [1]. Unfortunately, CHPT is not renormalizable at each order of perturbation, so one has to consider appropriate counterterms which depend on unknown coefficients. Gasser and Leutwyler [1] have analyzed all possible counterterms for the strong lagrangians to the next-to leading order $O(p^4)$ and they have calculated their coefficients fitting the experimental amplitudes. The coefficients of the counterterms depend on the scale μ used to renormalize the loop graphs. The authors of refs. [2, 3] have investigated the role of resonances in the strong chiral lagrangian and they have found that counterterms are saturated by resonance exchange.

The weak nonleptonic kaon decays were subject of interest in theoretical and experimental particle physics for almost forty years. The CHPT was applied to these processes [3, 4, 5, 6, 7, 13, 14, 15, 16, 22] but number of counterterms were found to be very large [4]. The vector-meson exchange contribution to $K \to \pi\pi\pi$ was studied within approach of [6, 7] and it was found that they change amplitudes by only few percent.

The scalar mesons were involved in chiral lagrangian in order to investigate coupling constants of the $O(p^4)$ [2, 3, 7]. Their treatment has been a persistent problem in the hadron spectroscopy [8, 9, 10] and therefore there are many different approaches developed in order to clarify presently confused nature of the known 0^{++} mesons [8]. Two best known scalar mesons $f_0(975)$ and $a_0(980)$ are very often treated as $q\bar{q}q\bar{q}$ states [9]. This interpretation was later reinvestigated within quark potential model as $K\bar{K}$ molecule [11]. Recent investigation, ref.[23], indicate that f_0 most probably is not $K\bar{K}$ molecule, nor an amalgam of two res-

onances [24], but a conventional Breit-Wigner-like resonance.

Fortunately, the CHPT does not recognize the nature of these resonances, but it gives a possibility to accommodate them as scalar octets mixed with scalar singlet.

We investigate this possibility motiveted by the fact that $f_0(975)$ and $K_0^*(1430)$ are effectively present in $K \to \pi\pi\pi$ decays, while $a_0(980)$ affects only the isospin-violating contribution to these amplitudes. Namely, the work of [2, 3, 7] is based on the accommodation of all scalars related CHPT couterterm parametars using only $a_0(980) \to \eta\pi$ decay. Both parts of the amplitude $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ are determined assuming vacuum-insertion approximation. We confirm the result that the $O(p^4)$ coorections induced by vector-meson exchange effective weak lagrangian do not contribute to $\Delta I = \frac{1}{2}$ part of the amplitude [6, 7]. We show that $\Delta I = \frac{3}{2}$ part of the amplitude coming from corresponding effecitive weak lagrangian is neither affected by vector mesons. But, scalar mesons affect both parts of the amplitude.

The factorization model (or vacuum insertion approximation)[12, 18, 19] which we use to determine the effective lagrangians which produce the CP conserving amplitudes, is formulated without any relations to resonances. However, the terms of the order of $O(p^4)$ [2, 3, 7] in the strong lagrangian are being saturated by resonance contributions, what implies that effective weak lagrangian of the same order $O(p^4)$ can be also influenced by their presence.

The outline of the work is following: in section 2 we repeat main features of the chiral lagrangian for strong and weak interactions, containing resonances. In the section 3 we derive and discuss the contributions to both parts of the amplitudes $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$.

2. $O(p^4)$ effective strong and weak lagrangians

The strong chiral lagrangian at the lowest order $O(p^2)$ is given by [1]

$$\mathcal{L}_s^2 \;=\; \frac{f^2}{4}\mathrm{tr}(D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U) \tag{1}$$

where

$$D_\mu U \;=\; \partial_\mu U + i(\mathrm{v}_\mu + \mathrm{a}_\mu)U + iU(\mathrm{v}_\mu - \mathrm{a}_\mu) \tag{2}$$

$$\chi = 2B_0(\mathrm{s} + i\mathrm{p}) \tag{3}$$

and $U = u^2$, is unitary 3×3 matrix, with $u = exp(-\frac{i}{\sqrt{2}}\frac{\Phi}{f})$, $\Phi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\varphi^i$, where ϕ is the matrix of the pseudoscalar fields. The external fields v_μ, a_μ, s, and p are hermitian 3×3 matrices in the flavor space. The parameters f and B_0 are the only free constants at $O(p^2)$, f is the pion constant in the chiral limit $f \simeq f_\pi$ and $B_0 =< 0|\bar{u}u|0 >$. The most general lagrangian of the order p^4 is given in the ref. [2]

$$\begin{aligned}
\mathcal{L}_4 \;=\; & l_1\mathrm{tr}(D_\mu U^\dagger D^\mu U)^2 + l_2\mathrm{tr}(D_\mu U^\dagger D^\nu U)\mathrm{tr}(D_\mu U^\dagger D^\nu U)\\
+\; & l_3\mathrm{tr}(D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U) + l_4\mathrm{tr}(D_\mu U^\dagger D^\mu U)\mathrm{tr}(\chi^\dagger U + \chi U^\dagger)\\
+\; & l_5\mathrm{tr}((D_\mu U^\dagger D^\mu U)(\chi^\dagger U + \chi U^\dagger)) + l_6(\mathrm{tr}(\chi^\dagger U + \chi U^\dagger))^2\\
+\; & l_7(\mathrm{tr}(\chi^\dagger U - \chi U^\dagger))^2) + l_8(\mathrm{tr}(\chi^\dagger U\chi^\dagger U + \chi U^\dagger \chi U^\dagger)\\
-\; & il_9\mathrm{tr}(F_R^{\mu\nu} D_\mu U^\dagger D^\nu U + F_L^{\mu\nu} D_\mu U^\dagger D^\nu U) + l_{10}(U_R^{\mu\nu} U F_{L\mu\nu})\\
+\; & h_1\mathrm{tr}(F_R^{\mu\nu}F_R^{\mu\nu} + F_L^{\mu\nu}F_L^{\mu\nu}) + h_2\mathrm{tr}(\chi^\dagger\chi)
\end{aligned} \tag{4}$$

where

$$F_{R,L}^{\mu\nu} = \partial^\mu(v^\nu \pm a^\nu) - \partial^\nu(v^\mu \pm a^\mu) - i[(v^\mu \pm a^\mu),(v^\nu \pm a^\nu)] \tag{5}$$

$l_1, ...l_{10}$ are ten real low-energy constants which together with f and B_0 completely determine the low-energy behavior of pseudoscalar meson interaction to $O(p^4)$. They arise at order p^4 and they are in general divergent (except l_3 and l_7) [1, 2, 3]. They absorb the divergences of the loops arising from $\mathcal{L}_2$. It is important to keep in mind that they depend on a renormalization scale μ, which does not show up

in observables. Following the work of [6] we simply introduce vector fields in the chiral lagrangian, even this is not the unique choice [2]:

$$\mathcal{L}_s(V) = -\frac{1}{4}\mathrm{tr}(\bar{V}_{\mu\nu}\bar{V}^{\mu\nu}) + \frac{1}{2}M_V^2\mathrm{tr}(\bar{V}_\mu - \frac{i}{g}\Gamma_\mu)^2 \tag{6}$$

Here

$$\bar{V}_{\mu\nu} = \hat{V}_{\mu\nu} - ig[\hat{V}_\mu, \hat{V}_\nu] + \frac{i}{4g}[u_\mu, u_\nu] + \frac{1}{2g}f_{\mu\nu} \tag{7}$$

$$\tag{8}$$

where $\hat{V}_{\mu\nu} = \nabla_\mu \hat{V} - \nabla_\nu \hat{V}_\mu$ and and ∇_μ is "covariant derivative"

$$\nabla^\mu X = \partial^\mu X + [\Gamma^\mu, X] \tag{9}$$

with

$$\Gamma^\mu = \frac{1}{2}\{u^\dagger[\partial^\mu - i(\mathrm{v}^\mu + \mathrm{a}^\mu)]u + u[\partial^\mu - i(\mathrm{v}^\mu - \mathrm{a}^\mu)]u^\dagger\} \tag{10}$$

The strength $\mathrm{f}_{\mu\nu} = u\mathrm{l}_{\mu\nu}u^\dagger + u^\dagger\mathrm{r}_{\mu\nu}$ with correspoding l_μ and r_μ determined as external gauge fields of $SU(3)_L \times SU(3)_R$ as $\mathrm{l}_\mu = \mathrm{v}_\mu - \mathrm{a}_\mu$ and $\mathrm{r}_\mu = \mathrm{v}_\mu + \mathrm{a}_\mu$. Following ref. [2, 3] from lagrangian (6) one dereives:

$$\mathcal{L}_s(V) = -\frac{1}{4}\mathrm{tr}(\hat{V}_{\mu\nu}\hat{V}^{\mu\nu}) + \frac{1}{2}M_V\mathrm{tr}[\hat{V}_\mu\hat{V}_\mu) \tag{11}$$

and for interacting fields

$$\begin{aligned}\mathcal{L}_{int}(V) &= -\frac{\sqrt{2}G_V}{4M_V}\mathrm{tr}(\hat{V}_{\mu\nu}[u^\mu, u^\nu]) - \frac{G_V}{\sqrt{2}M_V}\mathrm{tr}(f_{\mu\nu}\hat{V}^{\mu\nu}) \\ &- i\frac{G_V^2}{8M_V^2}\mathrm{tr}([\mathrm{u}^\mu, \mathrm{u}^\nu]\mathrm{f}_{\mu\nu}) + \frac{\mathrm{G}_\mathrm{V}^2}{8\mathrm{M}_\mathrm{V}^2}\mathrm{tr}([\mathrm{u}_\mu, \mathrm{u}_\nu][\mathrm{u}^\mu, \mathrm{u}^\nu])\end{aligned} \tag{12}$$

The relevant coupling constant entering into (12) are determined using decay widths $\Gamma(\rho \to e^+e^-)$ and $\Gamma(\rho \to \pi^+\pi^-)$. Namely, G_V is related to $\rho \to e^+e^-$ while F_V to $\rho \to \pi^+\pi^-$. The choice of lagrangian is not unique and these constants are in general independent [2, 3], but our particular choice satisfies the so-called the Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin relation[20, 21]

$$F_V = 2G_V = \frac{M_V}{\sqrt{2}g} \tag{13}$$

The kinetic term of the strong scalar lagrangian is given by

$$\mathcal{L}_k(S) = \frac{1}{2}\mathrm{tr}(\nabla_\mu S \nabla^\mu S - M^2 S^2) \tag{14}$$

where S is the scalar octet and M_S correspondes to the scalar masses in the chiral limit. For scalar singlet there is the kinetic term of lagrangian

$$\mathcal{L}_k(S_1) = \frac{1}{2}(\partial^\mu S_1 \partial_\mu S_1 - M_{S_1}^2 S_1^2) \tag{15}$$

The known scalar resonances f_0, a_0, K_0^* can be described as the linear combinations of octet and singlet states. For example a_0 and f_0 can be treated like ρ and ω vector mesons

$$f_0(975) = \frac{1}{\sqrt{3}}S_8 + \frac{2}{\sqrt{6}}S_1 \tag{16}$$

$$a_0(980) = -\frac{2}{\sqrt{6}}S_1 + \frac{1}{\sqrt{3}}S_8 \tag{17}$$

Their interactions with Goldstone pseudoscalars can be described writing the most general $SU(3)_L \times SU(3)_R$ lagrangian taking into account C and P properties of pseudoscalars and scalars [2, 3]

$$\mathcal{L}_{int}(S) = c_d \mathrm{tr}(S u_\mu u^\mu) + c_m \mathrm{tr}(S\chi_+) + \bar{c}_d S_1 \mathrm{tr}(u_\mu u^\mu) + \bar{c}_m S_1 \mathrm{tr}(\chi_+) \tag{18}$$

where

$$u_\mu = i u^\dagger D_\mu U u^\dagger \tag{19}$$

$$\chi_+ = u^\dagger \chi u^+ + u \chi^+ u \tag{20}$$

The experimental values of decay widths of f_0, a_0, K_0^* are given in Particle Data **92**

$$\Gamma(f_0 \to \pi\pi) = 36\mathrm{MeV} \tag{21}$$

$$\Gamma(a_0 \to \eta\pi) = 59\mathrm{MeV} \tag{22}$$

$$\Gamma(K_0^* \to K^-\pi^+) = 267\mathrm{MeV} \tag{23}$$

Assuming the $M_S \simeq M_{S_1}$ and fitting the experimental data for decay widths, we derive

$$c_d = 0.0220\text{GeV}, \ \ c_m = 0.0288\text{GeV} \tag{24}$$

and

$$\bar{c}_d = -0.0127\text{GeV}, \ \ \bar{c}_m = 0.0166\text{GeV} \tag{25}$$

These results are different then ones obtained in ref.[2] using a_0 decay only

$$\mid c_d \mid = 0.032\text{GeV}, \ \ \mid c_m \mid = 0.042\text{GeV} \tag{26}$$

$$c_d c_m > 0 \tag{27}$$

and

$$\bar{c}_d \ = \ \frac{\epsilon}{\sqrt{3}} c_d, \ \ \bar{c}_m = \frac{\epsilon}{\sqrt{3}} c_m \tag{28}$$

$$\epsilon = \pm 1 \tag{29}$$

obtained for large N_c limit.

Our calculation of the coupling parametars in interacting chiral lagrangian will lead to values of L_5 and L_8 by a factor ~ 2 smaller then those in references [2]. That will imply that these counterterms can be saturated by contributions coming from other resonances.

Following the work of Kambor et al.[4, 5] the CP conserving weak lagrangian can be written in the following form

$$\mathcal{L}_w \ = \ \frac{4c_2}{f^4}\text{tr}(\lambda_6 \mathcal{J}_\mu \mathcal{J}^\mu) + \frac{4c_3}{f^4} t^{jl}_{ik} \text{tr}(Q^i_j \mathcal{J}_\mu) \text{tr}(Q^k_l \mathcal{J}^\mu) \tag{30}$$

where $\mathcal{J}_\mu$ is the weak current determined by the lowest order expression of the left-handed current in the lagrangian (1). The couplings c_2 and c_3 are phenomenologically determined in [5, 6]. In addition to vector-meson exchange analyzed in [6, 7] there is a scalar- meson contribution. The weak current becomes

$$\mathcal{J}_\mu \ = \ u^\dagger \{ -\frac{f^2}{2} u_\mu - \frac{F_v M_v}{\sqrt{2}} V_\mu - c_d \{u^\mu, S\} - 2\bar{c}_d u^\mu S_1 \} u \tag{31}$$

This expression is obtained by isolating the terms linear in $v_\mu - a_\mu$.

In the further study of resonances influence on $K \to \pi\pi\pi$ decay amplitude we use the factorized form of the weak lagrangian. This procedure is equivalent to evaluation of the matrix elements of four-quark weak lagrangians in the vacuum-insertion approximation [12]. In our case using the analysis in ref [5], we take

$$\begin{aligned}\frac{c_2}{f^2} &= 6.6 \cdot 10^{-8} &(32)\\ \frac{c_3}{f^2} &= -8.3 \cdot 10^{-10} &(33)\end{aligned}$$

It is important to point out that the coupling constants of the chiral lagrangians are not fixed by chiral symmetry. In this case they are determined including next-to-leading order counterterms. The c_2/f^2 is reduced by 30% , while c_3/f^2 is unaffected by these corrections. The lagrangian given by factorization approximation describes the weak interaction of the pseudoscalars, vector-mesons and scalar-mesons. Eliminating vector and scalar mesons by strong interaction like in [2] we derive the following weak lagrangian containing effectively vector mesons

$$\mathcal{L}_w^8(V) = \frac{c_2}{M_V^2}\{\mathrm{tr}(\lambda_6 u^\dagger u_\mu u^\nu u_\nu u^\mu u) - \mathrm{tr}(\lambda_6 u^\dagger u_\mu u^\mu u_\nu u^\nu u)\} \tag{34}$$

and scalar mesons

$$\begin{aligned}\mathcal{L}_w^8(S) &= \frac{c_2 c_d}{M_S^2 f^2} 2\{c_d \mathrm{tr}(\lambda_6 u^\dagger \{u_\mu, \{u^\mu, u_\nu u^\nu\}\} u)\\ &+ c_m \mathrm{tr}(\lambda_6 u^\dagger \{u_\mu, \{u^\mu, \chi_+\}\} u) - \frac{4}{3} c_d \mathrm{tr}(u_\mu u^\mu) \mathrm{tr}(\lambda_6 u^\dagger u_\mu u^\mu u)\\ &- \frac{4}{3} c_m \mathrm{tr}(\chi_+) \mathrm{tr}(\lambda_6 u^\dagger u_\mu u^\mu u)\} + \frac{\bar{c}_d}{M_S^2 f^2} 2\{\bar{c}_d \mathrm{tr}(u_\mu u^\mu) \mathrm{tr}(\lambda_6 u^\dagger u_\mu u^\mu u)\\ &+ \bar{c}_m \mathrm{tr}(\chi_+) \mathrm{tr}(\lambda_6 u^\dagger u_\mu u^\mu u)\} &(35)\end{aligned}$$

The analysis of CP-invariant effective weak lagrangian transforming as $(27_L, 1_R)$ under $SU(3)_L \times SU(3)_R$ results in the following effective weak lagrangians

$$\mathcal{L}_w^{27}(V) = \frac{4c_3}{f^4} C_{ij}^{lk} (\mathcal{P}_\mu^3)_{ij} (u^\dagger u^\mu u)_{lk} \tag{36}$$

where

$$\mathcal{P}^3_\mu \quad = \quad -i\frac{G_V F_V}{2M_V}\nabla_\nu[u^\mu, u_\nu] \tag{37}$$

for parts containing vectors, and for part containing scalars

$$\begin{aligned}\mathcal{L}^{27}_w(S) \quad &= \quad -\{C^{lk}_{ik}\frac{2c_d}{M_S^2}\{u^\dagger\{u_\mu, j^S\}u - \frac{1}{3}\mathrm{tr}(j^S 2u^\dagger u_\mu u)\}_{ik}\mathcal{J}^\mu_{lk} \\ &+ \quad C^{ln}_{ik}\frac{c_d}{M_S^2}\mathcal{J}^\mu_{ik}\{u^\dagger\{u_\mu, j^S\}u - \frac{1}{3}\mathrm{tr}(j^S 2u^\dagger u_\mu u)\}_{ln} \\ &+ \quad C^{lk}_{ik}[\frac{2\bar{c}_d}{M_S^2}(u^\dagger u_\mu j^S u)]_{ik}\mathcal{J}^\mu_{lk} + C^{ln}_{ik}\frac{2\bar{c}_d}{M_S^2}\mathcal{J}^\mu_{ik}(u^\dagger u_\mu j^S u)_{ln}\}\end{aligned} \tag{38}$$

where j^S is defined as

$$j^S = c_d u_\nu u^\nu + c_m \chi_+ \tag{39}$$

and the constants C^{lk}_{ik} are determined as $C^{11}_{32} = C^{11}_{23} = 3$ and $C^{12}_{13} = C^{21}_{31} = 1$. These effective weak lagrangians are not the only source of resonance presence in $K \to \pi\pi\pi$. There are contributions coming from effective strong lagrangian of the $O(p^4)$ order which counterterms are saturated by vector and scalar-meson resonances given in the equation (4). These weak interactions occur only between pseudoscalar meson states. In order to have complete analysis we include in the calculation of $K \to \pi\pi\pi$ amplitude contributions coming from this lagrangian. The analysis of [7] considers the resonance contributions deriving the effective weak lagrangians within large-N limit approximation. Our result agrees with theirs within this limit.

3. Effective resonance contribution to the decomposed $K \to \pi\pi\pi$ amplitude

In the notation of the reference[5] we calculate resonance contribution using the isospin decomposition of $K \to \pi^+\pi^0\pi^-$ decay amplitude

$$\begin{aligned}A(K \to \pi^+\pi^0\pi^-) \quad &= \quad (\alpha_1 + \alpha_3) - (\beta_1 + \beta_3)Y \\ &+ \quad (\zeta_1 - 2\zeta_3)(Y^2 + \frac{X^2}{3}) + (\xi_1 - 2\xi_3)(Y^2 - \frac{X^2}{3})\end{aligned} \tag{40}$$

with $X = \frac{1}{m_\pi^2}(s_2 - s_1)$ and $Y = \frac{1}{m_\pi^2}(s_3 - s_0)$ where $s_i = (k - p_i)^2$, and $3s_0 = s_1 + s_2 + s_3$. Here k is kaon momentum, a p_i corresponds to momentum of i-th pion. The contributions coming from vector and scalar meson exchange give corrections to α_1, β_1, ζ_1, ξ_1, α_3, β_3, ζ_3, ξ_3

$$\delta\alpha_1 = \frac{c_2}{f^5 f_K M_S^2}[m_K^4(\frac{8}{27}c_d^2 + \frac{8}{9}\bar{c}_d^2 - \frac{4}{3}c_d c_m)] \tag{41}$$

$$\begin{aligned}\delta\beta_1 &= -\frac{c_2}{f^5 f_K M_S^2} m_\pi^2 m_K^2 (\frac{4}{9}c_d^2 + \frac{4}{3}\bar{c}_d^2 - 4 c_d c_m) \\ &- \frac{c_2}{f^2}\frac{2G_V}{M_V^2 f^3 f_K} m_K^2 m_\pi^2 \end{aligned}\tag{42}$$

$$\delta\xi_1 = \frac{c_2}{f^5 f_K M_S^2} m_\pi^4 (-\frac{2}{3}c_d^2 - 2\bar{c}_d^2) - \frac{c_2}{f^2}\frac{2G_V m_K^2}{M_V^2}\frac{m_\pi^4}{m_K^2 f^3 f_K}\frac{3}{2} \tag{43}$$

$$\delta\zeta_1 = \frac{c_2}{f^5 f_K M_S^2} m_\pi^4 (-\frac{2}{3}c_d^2 - 2\bar{c}_d^2) \tag{44}$$

$$\delta\alpha_3 = \frac{8c_3}{f^5 f_K M_S^2}[m_K^4(-\frac{1}{18}c_d^2 + \frac{5}{6}\bar{c}_d^2 + \frac{1}{3}c_d c_m)] \tag{45}$$

$$\begin{aligned}\delta\beta_3 &= -\frac{8c_3}{f^5 f_K M_S^2} m_\pi^2 m_K^2 (-\frac{1}{4}c_d^2 + \frac{5}{4}\bar{c}_d^2 - \frac{1}{4}c_d c_m) \\ &- \frac{3c_3}{f^2}\frac{2G_V}{M_V^2 f^3 f_K} m_K^2 m_\pi^2 \end{aligned}\tag{46}$$

$$\delta\xi_3 = -\frac{c_3}{2f^5 f_K M_S^2} m_\pi^4 (-3c_d^2 - 15\bar{c}_d^2) - \frac{c_3}{f^2}\frac{2G_V m_K^2}{M_V^2}\frac{9m_\pi^4}{2m_K^2 f^3 f_K} \tag{47}$$

$$\delta\zeta_3 = -\frac{c_3}{2f^5 f_K M_S^2} m_\pi^4 (-3c_d^2 - 15\bar{c}_d^2) \tag{48}$$

The analysis of the parameters α_1, β_1, ξ_1, ζ_1, α_3, β_3, ξ_3, and ζ_3 was first made by Develin and Dickey [22] and has been redone by Kambor et al [5], who have included $\Delta I = \frac{3}{2}$ corrections to X^2 and Y^2. These two fits are basically the

same. For the complete $O(p^4)$ corrections the loop contribution must be taken into the account. However, the inclusion of the loops results in dependence on the renormalization scale μ. We include in our numerical calculation results for loop contributions obtained first in ref. [4, 5, 7]. As it has been shown [6, 7] the $K \to \pi\pi\pi$ amplitude depends weakly on the choice of the μ scale used to renormalize the loop graphs.

In our calculation we make a difference between f (which is acctually$\simeq$ f_π) and f_K ($f_K \simeq$ 114MeV), though this difference is not determined by resonance counterterms [2, 3].

The numerical results are presented in the Tables 1 and 2. We denote as I the parametars regarding scalar mesons, determined in our approach - relations (24) and (25), while II denotes the set of parameters determined in the paper [2]. We take into account both contributions : the effective resonances exchange and the loops effect. In the Table 1 and 2 we give the results for $\mu = m_\eta$. The parameters determined by complete set of scalar meson decays influence the numerical values of isospin-decomposed $K \to \pi\pi\pi$ decay amplitude. The α_1 is still too small comparing the experimental fit, while β_1 is rather unchanged by scalar contribution. Even the overall corrections calculated using any choice of parameters are rather small, they cannot be neglected. The precise knowledge of parameters describing scalar mesons is necessary in order to better understand the role of scalar mesons in these decays, as well as in other weak, electromagnetic and strong processes.

Finally, we can comment on CP conserving decay $K_S \to \pi^+\pi^0\pi^-$ which is allowed even in the CP symmetry limit. This decay rate is determined by experimentally measured slope parameter γ_3 (see for example ref.[5, 13]) which in our calculation obtains the correction

$$\delta\gamma_3 \quad = \quad -\frac{4\sqrt{3}}{\sqrt{2}}\frac{c_3}{f^5 f_K M_S^2} m_\pi^2 m_K^2 (\frac{c_d^2}{6} + \frac{c_d c_m}{2}) \tag{49}$$

After performing the phase space integration we find

$$\Gamma(K_S \to \pi^+\pi^0\pi^-) = 1.69 \times 10^{-21}\text{GeV} \tag{50}$$

for the case of parameters in ref.[2], and for parameters we derived here, we find

$$\Gamma(K_S \to \pi^+\pi^0\pi^-) = 1.54 \times 10^{-21}\text{GeV} \tag{51}$$

In both cases decay width is two orders of magnitude larger than the CP-violating contribution arising mainly from $K_L - K_S$ mixing, which should be measurebale in the near future(see e.g.[13]).

The conclusions of our investigations can be summarized as follows

(i) The corrections coming from resonance exchange are rather small, they do depend on the choice of the parameters determined by scalar mesons data, which still cannot be definitely fixed due to lack of the experminetal data.

(ii) The analysis does not fully support phenomenological fit of [5] where "traces" of vector mesons in $K \to \pi\pi\pi$ amplitude are seen, since we found that scalar-meson corrections might be larger then vector ones.

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Amplitude	Tree	Loops	$I(S)$	$II(S)$	V	Tot$I$	TotII
α_1	7.80	2.52	-0.81	-0.37	0	9.51	9.94
β_1	-1.85	-1.07	0.24	0.11	-0.78	3.46	3.59
ζ_1	0	-0.03	-0.006	-0.0028	0	-0.09	-0.06
ξ_1	0	-0.12	-0.0059	-0.028	-0.09	-0.121	-0.118
α_3	52.07	-23.4	2.475	1.75	0	31.06	30.34
β_3	13.9	-2.28	-0.22	-0.10	-2.3	11.41	11.52
ζ_3	0	-0.0723	-0.0044	-0.0021	0	-0.076	-0.074
ξ_3	0	-0.126	0.0176	0.0083	-2.78	-0.12	-0.118

Table 1: All values are given in units $\frac{c_2}{f^2}$ for the corrections describing $\Delta I = \frac{1}{2}$ part of the amplitude and in units $\frac{c_3}{f^2}$ for the corrections coming from $\Delta I = \frac{3}{2}$ part of the amplitude. In the first column values coming from calculations of tree diagram, are given. In the second column there are loop corrections calculated in ref.[5] for the $\mu = m_\eta$. Third and fourth columns describe contributions of scalar meson resonances with fit from ref.[2] and for our fit. In the fifth column the contribution determined by vector mesons exchange is presented. The last two columns show the final corrections for two sets of parameters (ref.[2] and from our fit).

Amplitude	Fit	I	II
α_1	91.71	63.6	65.6
β_1	-25.68	-22.8	-23.7
ζ_1	-0.047	-0.059	-0.04
ξ_1	-0.151	-0.08	-078
α_3	-7.36	-2.58	-2.51
β_3	2.43	0.947	9.56
ζ_3	-0.021	-00063	-0.0061
ξ_3	-0.012	-0.09	-0.098

Table 2: All values are given in units 10^{-8}. In the first column we report values obtained in ref.[5] fitting the expermental data. In the second and third column complete results for $K \to \pi\pi\pi$ amplitude presented for the set of parameters in ref.[2] (I) and for the set of parameters derived in this paper (II) are given.

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